

# SCALABLE FEEDBACK ALGORITHMS FOR DISTRIBUTED TRANSMIT BEAMFORMING IN WIRELESS NETWORKS

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## ABSTRACT

We explore a class of techniques for distributed transmit beamforming where the beamforming target sends cumulative feedback that is broadcast to all of the beamforming nodes. The simplest technique in this class is a 1-bit RSS feedback algorithm that has been studied in detail in the literature. Under this 1-bit algorithm, transmitters make random phase perturbations and the receiver periodically sends 1 bit of feedback indicating whether the received signal strength has increased or not compared to what was observed in the past. While this simple algorithm has very attractive properties such as dynamic tracking of time-varying phases, robustness to noise and other disturbances and is also simple to implement, we show in this paper that it also has serious limitations such as slow convergence and poor tracking performance in the presence of frequency offsets between the transmitters. We then show that enhanced feedback algorithms where the receiver sends as feedback several bits of feedback indicating the amplitude and phase of the received signal over time, are able to achieve beamforming in the presence of frequency offsets and large feedback channel latencies, while retaining the scalability and robustness of the 1-bit algorithm.

**Index Terms**— distributed beamforming, cooperative transmission, virtual antenna arrays

## 1. INTRODUCTION

Distributed transmit beamforming (DTB) is a wireless transmission technique where a group of transmitters organize into a *virtual antenna array* and cooperatively transmit a common message signal to a distant receiver. This technique is attractive because it allows nodes with simple omnidirectional antennas to collaboratively obtain the directivity (and associated energy efficiency benefits) of antenna arrays. A cooperative array of  $N$  nodes can potentially achieve an energy efficiency of  $N$ ; thus for instance, a 10-node array can achieve the same SNR at the receiver with only a total power of  $\frac{1}{10}$  of the power required by a node transmitting individually.

The main challenge in realizing the large potential gains from distributed transmit beamforming is in precisely synchronizing the individual RF signals from each cooperating transmitter so that they are aligned in phase at the receiver. This is in contrast with cooperative *diversity* schemes [1] that do not require fine-grained synchronization, but deliver only diversity gains in fading channels rather than energy efficiency gains from beamforming.

The problem of synchronizing transmitters for distributed transmit beamforming has attracted a great deal of attention over the last decade (see the survey article [2]). A number of synchronization techniques have been developed, including full-feedback closed-loop [3], 1-bit closed-loop [4], master-slave open-loop [5] synchronization, round-trip synchronization [6], and two-way synchronization [7]. These techniques offer different sets of tradeoffs between simplicity, overheads associated with coordination messages between the transmitters, and overheads associated with feedback from the receiver.

The 1-bit feedback technique introduced in [4, 8] offers one example of this tradeoff. This algorithm has attractive properties of robustness to noise, estimation errors, and other disturbances and it dynamically adapts to channel time-variations. The 1-bit algorithm also has the very desirable property of *scalability*: the implementation of the algorithm does not depend on the number of collaborating transmitters; nodes can join and leave the virtual array at any time and the algorithm automatically adapts without any reconfiguration.

Finally the simplicity of this algorithm makes it possible to implement it on inexpensive hardware, and indeed distributed beamforming using variations of this basic algorithm has been demonstrated on multiple experimental prototypes [9, 10] at various frequencies. However, it has some serious limitations including slow convergence and, most notably, poor tracking performance in the presence of frequency offsets; this in practice this requires some explicit mechanism to ensure that the frequency offsets between the different transmitters is not too large.

These limitations follow from the very sparse amount of feedback provided by the receiver. This paper introduces an approach using greater amounts of feedback which retains the attractive features of the 1-bit algorithm while improving its

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convergence and tracking performance.

The rest of this paper is organized as follows. Section 2 motivates the paper with some analysis and experimental results illustrating the limitations of the 1-bit algorithm. Section 3 introduces a class of algorithms for distributed beamforming that uses rich feedback that addresses the limitations of the 1-bit algorithm. Numerical results showing the effectiveness of the proposed rich-feedback algorithms are presented in Section 4 and Section 5 concludes.

## 2. LIMITATIONS OF THE 1-BIT FEEDBACK ALGORITHM.

The 1-bit feedback algorithm is described as follows. In the  $m$ -th time-slot, the  $k$ -th transmitter randomly adjusts its phase from  $\phi_k[m]$  to  $\phi_k[m] + \Delta\phi_k[m]$ , where  $\Delta\phi_k[m]$  and  $\Delta\phi_l[m]$  are random and independent for  $k \neq l$ . The receiver monitors the received signal strength (RSS), and broadcasts a single bit of feedback to the transmitting nodes indicating whether the RSS has increased. The transmitters all retain the phase perturbations if the RSS has increased and discard otherwise.

The transmitters converge to coherence *almost surely* under mild conditions on the distribution of the  $\Delta\phi_k$ . Further, the algorithm is robust to noise, estimation errors, lost feedback signals and time-varying phases, is scalable and easy to implement. Yet this algorithm has a number of shortcomings:

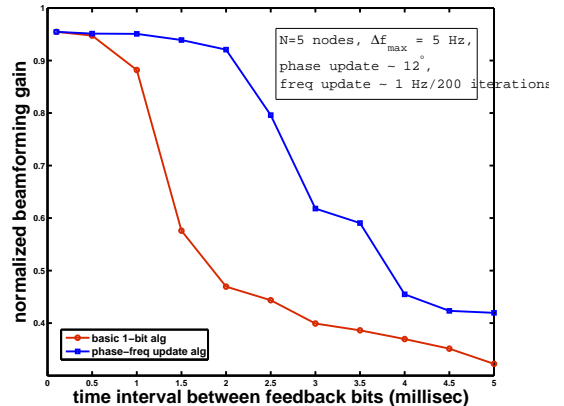
1. **Slow convergence rate.** While the time to convergence of the 1-bit algorithm (when properly optimized, increases linearly with number of transmitters [11]), in absolute terms, it requires a large number of time-slots.
2. **Latency limitations.** The algorithm neglects latency in the feedback channel; it assumes that the feedback signal is available instantaneously and simultaneously at all the transmitting nodes. Latency makes maintaining time-slots across the beamforming nodes a challenge, requiring longer time-slots, that slow convergence further.
3. **Poor performance with frequency offsets.** The simple version of the 1-bit algorithm assumes the transmitting nodes are already frequency-locked and thus have a fixed (but unknown) phase relationship. Non-zero frequency offsets between transmitters manifest themselves as rapid time-variations in the phase. While variations of the 1-bit algorithm have been developed that can handle frequency offsets [10], these too require high feedback rates on the order of  $100 \times \Delta f_{max} \times N$ , where  $N$  is the number of transmitting nodes and  $\Delta f_{max}$  is the maximum frequency offset between the transmitters.

To amplify the limitations of [10] we observe that in addition to phase updates it also randomly increments the frequencies as  $\omega_k[m] + \mu\Delta\phi_k[m]$ , with  $\mu > 0$ . This renders the vector of frequency updates parallel to that of the phase updates, leading to a severe lack of *persistent excitation*, [12]. The

effect of this is that barring nongeneric initial phase and frequency offsets, exact synchronization in both frequency and phase is impossible. In principle this is remedied by choosing the phase and frequency offsets as:  $\phi_k[m] + \Delta\phi_{k1}[m] + \Delta\phi_{k2}[m]$ , and  $\omega_k[m] + \mu_1\Delta\phi_{k1}[m] + \mu_2\Delta\phi_{k2}[m]$ , with  $\mu_1 \neq \mu_2$  and  $\Delta\phi_{k1}[m]$ ,  $\Delta\phi_{k2}[m]$  mutually independent. This however, slows down convergence further, as now two independent random perturbations must together produce favorable net change, as opposed to the more likely event of a single random perturbation doing so.

These limitations were apparent in a recent series of over-the-air experiments at Raytheon BBN Technologies. Three transmitter nodes were initially configured to derive their RF carriers as well as baseband clocks from a common reference clock. The receiver broadcast 1-bit SNR feedback to the transmitter using UDP packets on an Ethernet interface. The latencies of the Ethernet MAC as well as the delays in sending the UDP packets and waiting for ACKs from each node limited the overall feedback rate to around 10 Hz. The 1-bit algorithm was able to provide near optimal beamforming gains in this setup. However, the beamforming quickly deteriorated when the transmitters were driven by separate reference oscillators. The RF carriers derived from high quality ovenized crystal oscillators [13] differed by several hertz and the 1-bit feedback technique could not converge.

These observations are illustrated by simulations in Fig. 1; this plot shows the normalized beamforming gain (which equals 1 for perfect coherence) as a function of time between feedback messages for both the basic 1-bit feedback algorithm and its variant described in [10] that also does frequency adjustments. It is clear from this plot that the beamforming gain quickly deteriorates for feedback rates less than  $\frac{1}{2.5 \text{ ms}} = 40 \text{ Hz}$  for an array of 5 transmitters with maximum frequency offsets of 5 Hz.



**Fig. 1.** Performance of the 1-bit algorithm and its variants in the presence of frequency offsets between transmitters.

### 3. SCALABLE ALGORITHMS USING RICH FEEDBACK

Motivated by the constraints of the experimental setup described above, we consider the more general class of algorithms shown in Fig. 2.

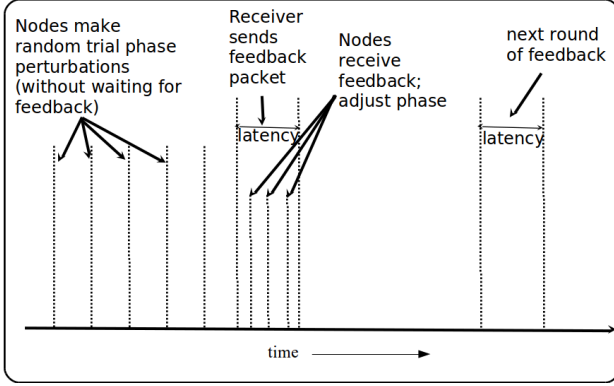


Fig. 2. Generalized class of feedback algorithms.

Consider a setup with  $k = 1 \dots N$  transmitters in the distributed array. For simplicity let us assume constant time-slots of duration  $T_{slot}$ , though this algorithm can be generalized to work without any time-slotting. As shown in Fig. 2, at the beginning of each time-slot, all transmitting nodes independently apply a sequence of random phase perturbations to their transmitted signal. Specifically let  $t = 0$  indicate the beginning of a time-slot; transmitter  $k$  randomly picks a set of perturbations  $\Delta\phi_k[m]$ ,  $m = 0 \dots M - 1$ , and the complex amplitude of the resulting signal at the receiver is  $r(t) = \sum_{k=1}^N a_k \exp(j(\phi_k + \Delta\phi_k[m]))$  for  $t \in [m\Delta T, (m+1)\Delta T]$ .

The total duration of the estimation phase of the time-slot is  $T_{est} \doteq M \times \Delta T$ , and the “duty cycle”  $\frac{T_{est}}{T_{slot}}$  of the estimation process can be quite small (e.g. 10%). Note that this duty cycle is independent of  $N$ , thus assuring the scalability of the algorithm. We assume that the frequency offsets between the transmitters are small enough that all the received phases  $\phi_k$  are approximately constant over the estimation time  $T_{est}$ ; in other words, we assume that the maximum frequency offset  $\Delta f_{max} \ll \frac{1}{T_{est}}$ .

The feedback signal from the receiver at the end of the time-slot basically contains the complex amplitude of the received signal  $r(t)$ ,  $t \in [0, T_{est}]$ . If the phase perturbations are small i.e.  $\Delta\phi_k[m] \ll 1$ , we can define

$$\exp(j(\phi_k + \Delta\phi_k[m])) \approx \exp(j\phi_k)(1 + j\Delta\phi_k[m]) \quad (1)$$

This leads to

$$r(t) = \tilde{r} + \sum_{k=1}^N j a_k \exp(j\phi_k) \Delta\phi_k[m] \quad (2)$$

where the “mean received amplitude”  $\tilde{r} \equiv \sum_{k=1}^N a_k \exp(j\phi_k)$  can be estimated as  $\tilde{r} \frac{1}{T_{est}} \int_0^{T_{est}} r(t) dt$ , where we assumed that the phase perturbations are chosen from a zero-mean distribution and therefore their effects cancel out when averaged over the estimation interval.

Using (2) and noting that the phase perturbations are uncorrelated across different nodes, we can write

$$\begin{aligned} \tilde{y}_k &\doteq \sum_{m=0}^{M-1} ((r(m\Delta T) - \tilde{r}) \Delta\phi_k[m]) \quad (3) \\ &= j a_k \exp(j\phi_k) \sum_{m=0}^{M-1} (\Delta\phi_k[m])^2 \\ &+ \sum_{l \neq k} j a_l \left( \sum_{m=0}^{M-1} \Delta\phi_k[m] \phi_l[m] \right) \\ &= j \exp(j\phi_k) a_k C_{kk} + \sum_{l \neq k} j \exp(j\phi_l) a_l C_{lk} \quad (4) \end{aligned}$$

where  $C_{kk} \doteq \sum_{m=0}^{M-1} (\Delta\phi_k[m])^2 \approx M \sigma_{\Delta\phi}^2 a_k (j \exp(j\phi_k))$  and the “cross-correlation”  $C_{lk}$  is a zero mean random variable. Thus, we can use  $\tilde{y}_k$  for an estimate  $\tilde{\phi}_k \doteq \angle(-j\tilde{y}_k)$  of  $\phi_k$ , where the terms involving  $C_{kl}$  can be considered as a random variable representing the estimation error.

The transmitter  $k$  can then use this phase estimate  $\tilde{\phi}_k$  to compensate for its phase offsets with respect to the other nodes in the next time-slot. However, this raw phase estimate can be quite noisy as noted above due to interference from other transmitters. Therefore we use a simple 2-state Kalman filter to “smooth” out these estimates over many estimation time-slots.

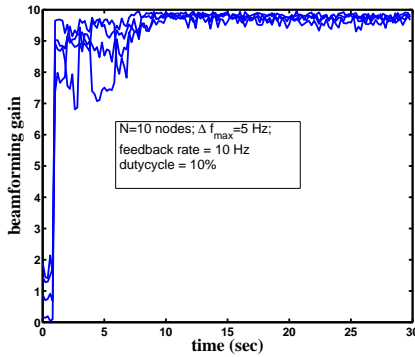
The Kalman filter is based on the following 2-state state-space model that has received a lot of attention for modeling oscillator dynamics [14], and for synchronization protocols in IEEE 1588 [15].

$$x[k+1] = Fx[k] + w[k]; \quad y[k] = Hx[k] + v[k] \quad (5)$$

Here phase and frequency offsets (with respect to the phase of the receiver’s oscillator) are the two state variables,  $w \sim N(0, C)$  is the white process noise, the observation  $y$  is a noisy measurement of the phase, so that  $H = [1, 0]$  and  $v \sim N(0, R)$  is a white measurement noise. The correlation matrix  $C$  can be estimated by fitting the experimental measurements of the Allan variance [14] to the theoretical variance of (5) and the measurement noise variance is set as  $R = \frac{M}{N-1}$ .

### 4. PERFORMANCE OF THE RICH FEEDBACK ALGORITHM

The performance of this algorithm is illustrated in Fig. 3 which shows 4 simulation runs of the beamforming gain as a function of time. It can be seen that the algorithm quickly converges to near-optimal beamforming gains even with large frequency offsets and a slow feedback rate.



**Fig. 3.** Performance of beamforming algorithm using rich feedback.

## 5. CONCLUSION

In this paper, we showed that a rich feedback based approach can be used to develop scalable algorithms for distributed transmit beamforming. This approach generalizes a simple 1-bit feedback algorithm, and is able to achieve near-optimal beamforming gains even with significant frequency offsets and slow feedback rates. This suggests many promising areas for future work including an analytical study of the performance of this class of algorithms and their convergence properties, optimal design of phase perturbations, and extending this approach to more advanced virtual array applications such as steering nulls, multiple beams and so on.

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