Distributed Beamforming using 1 Bit Feedback: from Concept to Realization

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***Abstract*— We present theoretical and experimental results for a distributed beamforming system based on a simple 1-bit feedback algorithm. The algorithm is based on an iterative procedure, that synchronizes multiple transmitters to cooperatively send a common message signal coherently to a receiver, using only a single bit of feedback in each timeslot. Under this scheme, the transmitters make independent, random phase adjustments every timeslot, and retain only those phase adjustments that increase the SNR at the receiver. We describe the design of an experimental prototype to implement the beamforming algorithm, and present measurement data that shows the SNR gains from beamforming. We also analyze the convergence behavior of the procedure mathematically using a statistical approach. We show that the mathematical model gives accurate predictions for the convergence rate in static and time-varying channels, and use the analysis to demonstrate the scalability of the algorithm.**

I. INTRODUCTION

Cooperative communication has recently been investigated [1] as a method for improving the performance of wireless networks. The basic idea is for two or more radios to cooperatively form a *virtual antenna array*, and obtain diversity gains against channel fading, and array gains from increased directivity. Depending on the specific communication constraints, different combinations of distributed coding and array processing [2] have been proposed for cooperative communication. This paper describes a method for *distributed beamforming*, where a network of multiple transmitters cooperate to transmit a common message signal, *coherently* to a distant Base Station receiver (BS) in order to obtain diversity and array gains.

The potential benefits of distributed beamforming are well-known - full diversity and N-fold increase in energy efficiency for N transmitters. The main challenges are in synchronizing the RF carriers of all the transmitters, and measuring the channel of each individual transmitter to the BS. In the context of large-scale wireless networks, it would be advantageous to develop methods that scale well with the number of transmitters. This problem was investigated in [3], [4], which also described a procedure for distributed synchonization that was designed to minimize the amount of coordination with the BS. The basic idea is to first frequency-lock the carrier signals of all the transmitters using a master-slave architecture for the network, and then performing phase calibration and channel estimation to achieve coherent transmission. It was shown in [5] that a simple iterative procedure can be used for phase calibration and channel estimation, provided that the BS is able to send a single bit of feedback every timeslot.

The 1-bit feedback procedure of [5] has been shown [6] to have attractive performance and scalability properties. In this paper we present new theoretical results, and empirical observations from an experimental prototype of a distributed beamforming system based on this algorithm. The experimental results show that substantial portion of the theoretical SNR gains from beamforming are achievable in practical conditions. The theoretical model is based on an elegant statistical approach, and reveals interesting mathematical properties of the algorithm, when the number of transmitters is large.

The research described in this paper is related to previous work in many different areas. In closely related work, an iterative algorithm for beamforming was presented in [7], but for a centralized antenna array. Other authors have studied issues related to distributed beamforming [8], however most of the prior work in this area does not address the synchronization problem. There is also a substantial amount of literature on the theory of stochastic approximations; the 1-bit feedback algorithm described in this paper could be considered as a distributed version of the procedure described in [9]. To the best of our knowledge the prototype described in this paper is the first experimental demonstration of a cooperative wireless system.

The rest of the paper is organized as follows. Section II presents a communication model for the distributed beamforming system, and reviews the 1-bit feedback algorithm. An analytical study of the algorithm is presented in Section II-B. Section III motivates and describes the design of an experimental prototype to study the performance of this system. The theoretical results of Section II-B are extended to time-varying channels and compared with experimental measurements and simulation data in Section IV. Section V concludes with a brief discussion of future directions.

II. BACKGROUND

Fig. 1 shows our communication model for distributed beamforming. It consists of a network of N transmitters, that cooperatively transmits a common (complex baseband)

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message signal $m(t)$ by modulating it with its RF carrier signal. All transmitters are frequency locked to a reference carrier signal using the master-slave architecture described in [3]. As a result, all the carrier signals are at the same frequency f_c and there is no frequency offset between the transmitters. There is however, an arbitrary phase difference between the transmitters because of unknown propagation delays in the master-slave architecture. Thus we can write the carrier signal of transmitter i as:

$$c_i(t) = \Re\left(e^{j(2\pi f_c t + \gamma_i)}\right) \tag{1}$$

where γ_i is the phase offset. Transmitter i rotates the message signal by a angle θ_i, modulates it with $c_i(t)$ and transmits the modulated signal $s_i(t) = \Re\left(m(t)e^{j\theta_i}e^{j2\pi f_c t + j\gamma_i}\right)$ on the wireless channel. If its channel to the BS is $h_i = a_i e^{j\psi_i}$, then the total received signal at the receiver is:

$$r(t) = \Re\left(m(t)e^{j2\pi f_c t}\sum_{i=1}^{N} a_i e^{j\gamma_i + j\theta_i + j\psi_i}\right) \tag{2}$$

In practice, this signal at the receiver would be corrupted by additive noise which is not shown explicitly in (2). Let us denote the phase of the received signal from transmitter i as $\Phi_i = \gamma_i + \theta_i + \psi_i$. The power of the received signal is given by:

$$S_r = \Big|\sum_i a_i e^{j\Phi_i}\Big|^2 \tag{3}$$

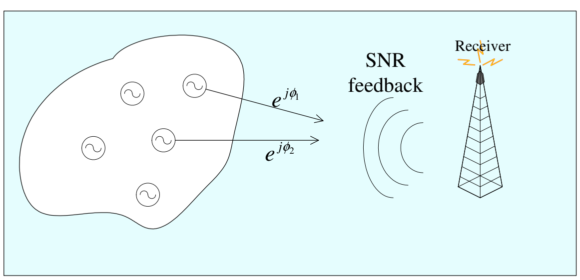

Fig. 1. Phase synchronization using receiver feedback

The power S_r is maximized when the signals from all the transmitters are received coherently at the BS, i.e. $\Phi_i =$ constant. The purpose of the distributed beamforming algorithm is for the transmitters to adjust their phase rotation θ_i to achieve this condition of coherence. We observe that if the BS is far away compared to distances between the transmitters, the channel attenuations a_i are of the same order of magnitude, and can be modeled as iid realizations of a stochastic fading process e.g. Rician fading. Without loss of generality, we normalize the signal units such that $E(a_i) = 1$. Similarly the received phase angles Φ_i are completely unknown before the feedback control algorithm is executed, and the initial values of Φ_i are modeled as uniformly distributed random variables in $(-\pi, \pi]$.

A. Review of Feedback Control algorithm for static channels

We now present a brief summary of the 1-bit feedback algorithm for distributed beamforming first proposed in [5].

1) Each transmitter keeps a record $\theta_{best,i}[n]$ of the best known value of its phase rotation, where n is the timeslot index. The BS measures the received signal strength (RSS) $\mathcal{Y}[n]$ at each timeslot n and keeps a record $\mathcal{Y}_{best}[n]$ of its best previously observed RSS:

$$\mathcal{Y}_{best}[n] = \max_{m \le n} \mathcal{Y}[m]$$
$$\mathcal{Y}[m] = \Big|\sum_{i=1}^{N} a_i e^{j\Phi_i[m]}\Big| \tag{4}$$

2) At timeslot $n+1$, each transmitter generates a random phase perturbation δ_i from some probability distribution $f_\delta(\delta_i)$ (this distribution can change in time), and transmits its message signal with an incremental phase rotation δ_i: $\theta_i[n+1] = \theta_{best,i}[n] + \delta_i$. This results in the received phase:

$$\Phi_i[n+1] = \Phi_{best,i}[n] + \delta_i \tag{5}$$

where $\Phi_{best,i}[n] \doteq \gamma_i + \theta_{best,i}[n] + \psi_i$ is the phase of the received signal from transmitter i corresponding to $\theta_{best,i}[n]$. Note that the received phases Φ_i and $\Phi_{best,i}[n]$ are auxiliary variables used in our analysis, and are not known to either the transmitters (which can only control the phase rotations θ_i and $\theta_{best,i}[n]$), or the BS (which can only observe the aggregate received signal $r(t)$).

3) The BS measures the received signal strength and generates a single bit of feedback that is set to '1' if the received signal strength in the current timeslot is better than the previous best RSS, and '0' otherwise. The BS then broadcasts this bit of feedback (the broadcast is assumed to be noiseless, which can easily be assured using large amounts of redundancy if needed).

4) The BS updates its value of $\mathcal{Y}_{best}[n+1]$ and the transmitters update the phase rotations $\theta_{best,i}[n+1]$ to retain the perturbations δ_i if the feedback bit is '1', and discard them otherwise.

5) The process is repeated in the next timeslot.

The update process can be written mathematically as:

$$\mathcal{Y}_{best}[n+1] = \begin{cases} \mathcal{Y}[n+1], & \mathcal{Y}[n+1] > \mathcal{Y}_{best}[n] \\ \mathcal{Y}_{best}[n], & \text{otherwise.} \end{cases} \tag{6}$$

$$\theta_{best,i}[n+1] = \begin{cases} \theta_{best,i}[n] + \delta_i[n], & \mathcal{Y}[n+1] > \mathcal{Y}_{best}[n] \\ \theta_{best,i}[n], & \text{otherwise.} \end{cases}$$

B. Analytical Model for Convergence

Fig. 2 shows the convergence behavior of the phase angles in a simulation of the 1-bit feedback algorithm. $\theta_{best,i}[n]$ and $\mathcal{Y}_{best}[n]$ are stochastic processes, whose behavior over time depends on the random choices δ_i. We'd like to model the behavior of this algorithm analytically to obtain insight into its performance, and also to maximize the convergence rate by choosing the distributions $f_\delta(\delta_i)$ optimally over time. Such an analytical model was derived in [6], and has been shown to give accurate results, when compared with simulations. We now sketch the details of this model.

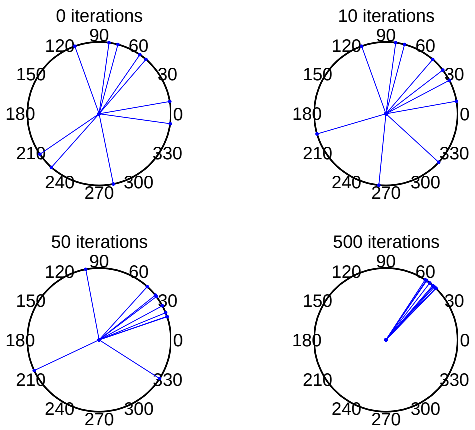

Fig. 2. Convergence of feedback control algorithm

The analysis is based on the empirical observation that the random process $\mathcal{Y}_{best}[n]$ evolves in a highly predictable way. For instance, Fig. 3 shows two simulated instances of the feedback algorithm, and we can see that the convergence of the two instances are close to each other. Therefore we examine the convergence behavior of the average $y[n]$ of the process $\mathcal{Y}_{best}[n]$ and use it to understand the behavior of the algorithm and optimize the convergence. $y[n]$ is formally defined as follows:

$$\begin{aligned} y[1] &= E\big(\mathcal{Y}_{best}[1]\big) \\ y[n+1] &= E\big(\mathcal{Y}_{best}[n+1]\big|\mathcal{Y}_{best}[n]=y[n]\big) \end{aligned} \quad (7)$$

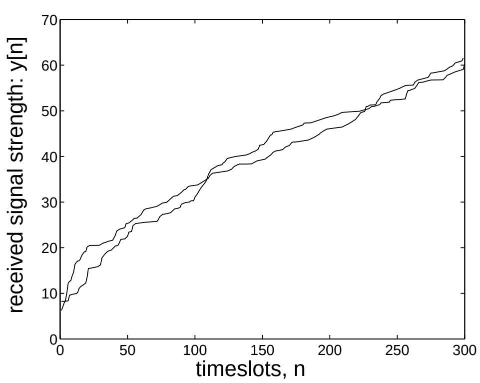

Fig. 3. Motivating the Analytical Model: two simulated instances with $N = 100, f_\delta(\delta_i) \sim \text{uniform}[-\frac{\pi}{20}, \frac{\pi}{20}]$.

Note that (7) defines $y[n]$ iteratively in time: $y[n+1]$ is evaluated as a conditional expectation of $\mathcal{Y}_{best}[n+1]$ given $\mathcal{Y}_{best}[n] = y[n]$. If we know the statistics of $\mathcal{Y}[n+1]$, we can compute the expectation in (7) using (6). $\mathcal{Y}[n+1]$ can be written using (4) and (5) as:

$$\mathcal{Y}[n+1] = \left|\sum_{i=1}^{N} a_i e^{j\Phi_{best,i}[n]+j\delta_i}\right| \quad (8)$$

The cumulative effect of the phase perturbations δ_i is to modify the complex received signal by increasing or decreasing its length and by rotating it. This is shown in Fig. 4, where x_1 and x_2 are zero mean random variables:

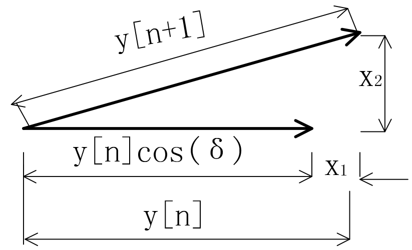

Fig. 4. Cumulative effect of phase perturbations on the received signal

$$\mathcal{Y}[n+1] = \left|C_\delta \mathcal{Y}_{best}[n] + x_1 + jx_2\right| \quad (9)$$

where C_δ is a constant whose precise definition will be given shortly. x_1 and x_2 can be expressed as the sum of iid contributions from each transmitter's perturbation δ_i. Therefore for large N, they look approximately Gaussian and their variances are:

$$\sigma_1^2 = \frac{1}{2}(1 - C_\delta^2)\sum_{i=1}^{N} a_i^2 - \frac{1}{2}(C_\delta^2 - C_{2\delta})\sum_{i=1}^{N} a_i^2\cos(2\phi_i) \quad (10)$$

$$\sigma_2^2 = \frac{1}{2}(1 - C_\delta^2)\sum_{i=1}^{N} a_i^2 + \frac{1}{2}(C_\delta^2 - C_{2\delta})\sum_{i=1}^{N} a_i^2\cos(2\phi_i) \quad (11)$$

where $\phi_i = \Phi_{best,i} - \Phi_0$,

$$\text{and } \Phi_0 \doteq \angle\Big(\sum_{i=1}^{N} a_i e^{j\Phi_{best,i}}\Big) \quad (12)$$

In the preceding equations, $C_\delta = E(\cos\delta_i)$ and $C_{2\delta} = E(\cos(2\delta_i))$, and both constants are parameters of the distribution function $f_\delta(\delta_i)$.

Using this representation of $\mathcal{Y}[n+1]$, we can evaluate the expectation in (7). We note however that this expectation depends on the distribution function $f_\delta(\delta_i)$ (through C_δ and $C_{2\delta}$), and also on the actual values of the received angles $\Phi_{best,i}$. However, for a large number of transmitters N, it is possible to simplify this considerably by using a statistical approach. The idea of the statistical model is subtle, therefore we discuss it in some detail next.

C. Statistical Model for the Received Phase Angles

This approach is motivated by the techniques of statistical physics. Consider a system consisting of a large number of non-interacting ideal gas particles (atoms). The macroscopic properties e.g. pressure of this system can be calculated using classical mechanics if the position and velocity of each atom was specified at some instant of time. However this approach requires the specification of 6 coordinates for each of (the order of) 10^{23} particles, and is therefore unsatisfactory and not very useful. The statistical physics approach is to use symmetry arguments and inquire into the *distribution* of the positions and velocities rather than their precise values. In the terminology of statistical physics, we are only interested in characterizing the *macrostates* of the system, each macrostate consisting of a huge number of microstates obtained by permutation of the coordinates of different atoms.

The key idea is that there exists one macrostate that contains almost all of the possible microstates [10]. Thus for instance, using the statistical model, we assert that the gas atoms are uniformly distributed in the volume of the system, the direction of their velocities is isotropic i.e. equal number of atoms are moving in all directions, and the fraction of the atoms with a speed, v is proportional to $e^{\frac{-mv^2}{2k_BT}}$, where T is the temperature of the system, m is the mass and k_B is the Boltzmann constant. Even though there are an infinite number of other possible positions and velocities (i.e. *microstates*) that are consistent with the constraints of this system, the overwhelming majority of those positions and velocities follow the distributions above, which may be regarded as "typical". The validity of this model is demonstrated by comparing its predictions with experiment.

We adopt a similar approach in modeling the received phases Φ_i. Initially, all the phases $\Phi_i[0]$ are completely unknown, and they can be considered as iid and uniformly distributed in $(-\pi, \pi]$. Even though any combination of the phases $\Phi_i[0]$ is equally likely, when the number of transmitters N is large, the empirical distribution of the angles $\Phi_i[0]$ is with high probability close to the distribution uniform$(-\pi, \pi]$. As the feedback algorithm progresses, the phases $\Phi_i[n]$ are no longer distributed independently of each other, but rather tend to get increasingly clustered. However for large N, we still expect the empirical distribution of $\Phi_i[n]$ to be close to some fixed distribution with high probability (analogous to the Maxwell-Boltzmann distribution for the velocities of ideal gas particles). Our goal is to find this distribution.

Recall that our definition of $y[n+1]$ in (7) is an average conditioned on $\mathcal{Y}_{best}[n] = y[n]$. This can be written as:

$$\mathcal{Y}_{best}[n] \equiv \Big|\sum_{i=1}^{N} a_i e^{j\Phi_{best,i}}\Big| = y[n] \tag{13}$$

$$\text{or} \quad \sum_{i=1}^{N} a_i \cos(\Phi_{best,i} - \Phi_0) = y[n] \tag{14}$$

The phase Φ_0, the phase of the total received signal (as defined in (12)), is of no interest to us, therefore we consider the distribution of the phases Φ_i *about* the overall phase Φ_0. In other words, we are interested in the distribution of $\phi_i = \Phi_{best,i} - \Phi_0$. We now rewrite the LHS of (14) as:

$$\sum_{i=1}^{N} a_i \cos\phi_i \equiv N \cdot E\big(a_i \cos\phi_i\big) \tag{15}$$

Note that we expressed the sum of the terms $a_i \cos\phi_i$ in terms of an expectation over the probability distribution of a_i and ϕ_i. As long as the expectation in (15) is carried out over the actual empirical distribution of $a_i \cos\phi_i$ (i.e. the relative frequencies of $a_i \cos\phi_i$ over its range), (15) is trivially true, because it is just the definition of the empirical average of $a_i \cos\phi_i$. For large N, however we can use typicality arguments to *estimate* the empirical distribution $f_\phi(\phi_i)$ without having to know the individual phase angles ϕ_i. Using $E(a_i) = 1$ and (15), the condition $\mathcal{Y}_{best}[n] = y[n]$ can be written as:

$$E\big(\cos\phi_i\big) = \frac{y[n]}{N} \tag{16}$$

This is the huge simplification offered by the statistical method, and it allows us to compute the variances in (10) and (11) as:

$$\sigma_1^2 = \frac{\alpha N}{2}\Big(\big(1 - C_\delta^2\big) - \big(C_\delta^2 - C_{2\delta}\big)E\big(\cos(2\phi_i)\big)\Big) \tag{17}$$

$$\sigma_2^2 = \frac{\alpha N}{2}\Big(\big(1 - C_\delta^2\big) + \big(C_\delta^2 - C_{2\delta}\big)E\big(\cos(2\phi_i)\big)\Big) \tag{18}$$

where $\alpha = E(a_i^2)$ is just a constant. For the simplest case that all transmitters have LoS channels of equal attenuation to the receiver, $\alpha = 1$.

All that remains is to compute $E\big(\cos(2\phi_i)\big)$. We compute this, by first deriving an estimate $f_\phi(\phi_i)$ of the empirical distribution of ϕ_i that satisfies the constraint (16). We now present the typicality argument to compute $f_\phi(\phi_i)$. Although many possible ϕ_i combinations can satisfy the constraint (16), the overwhelming majority of those ϕ_i combinations have the same empirical distribution if the number of transmitters N is very large. In order to calculate this typical distribution, we use the Conditional Limit Theorem from the theory of typical sequences ([11], Sec. 12.6). This theorem states that the empirical distribution of symmetrically distributed random variables conditioned on some symmetric constraints on the random variables, is, with high probability, the distribution that satisfies the constraints, and is closest in Kullback-Liebler distance from the unconditional distribution. In our case, the unconditional distribution of the phases ϕ_i is $u(\phi) \equiv$ uniform$(-\pi, \pi]$, because without any constraints, the phases ϕ_i will be uniformly distributed over $(-\pi, \pi]$. Finding the closest $f_\phi(\phi_i)$ is a problem in calculus of variations. To solve this, we construct a Lagrangian for this optimization problem as:

$$\begin{aligned} L\big(f_\phi(\phi_i)\big) &= \int_{-\pi}^{\pi} f_\phi(\phi_i)\log\Big(\frac{f_\phi(\phi_i)}{u(\phi_i)}\Big) \\ &\quad + C_1\Big(\int_{-\pi}^{\pi}\cos\phi_i f_\phi(\phi_i)d\phi_i - \frac{y[n]}{N}\Big) \\ &\quad + C_2\Big(\int_{-\pi}^{\pi} f_\phi(\phi_i)d\phi_i - 1\Big) \end{aligned} \tag{19}$$

We can show that the solution to (19) is as follows:

$$f_\phi(\phi_i) = \frac{1}{J_0(\lambda)}e^{\lambda\cos\phi_i} \tag{20}$$

where λ is the solution to $\frac{J_1(\lambda)}{J_0(\lambda)} = \frac{y[n]}{N}$ and $J_k(\lambda)$ is the modified Bessel function of the first kind and order k.

Fig. 5 shows a comparison of the distribution from (20) and an empirical distribution of the received phase angles ϕ_i obtained by simulation. The same figure also shows the Laplacian and Gaussian distributions for the phase angles for comparison. The close match between the empirical

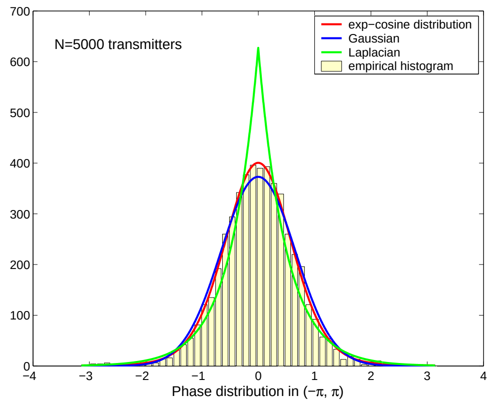

Fig. 5. Histogram of empirically observed phase angles compared with analytically computed distributions

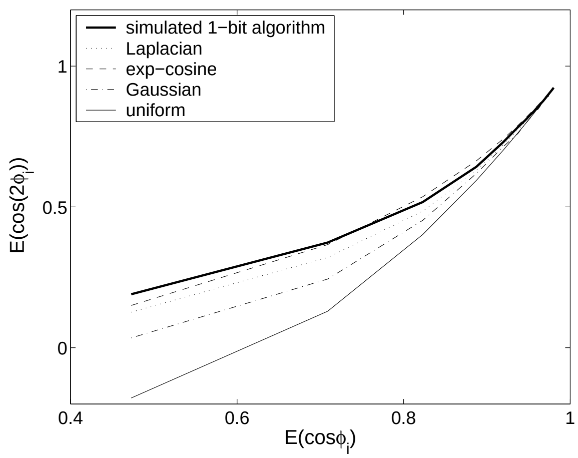

Fig. 6. Comparison of $E\big(\cos(2\phi_i)\big)$ from empirically observed phase angles with analytically computed distributions

histogram and the distribution in (20) (the "exp-cosine" distribution) observed in Fig. 5 validates our analysis. For the distribution in (20), we can show:

$$E\big(\cos(2\phi_i)\big) \equiv \frac{J_2(\lambda)}{J_0(\lambda)} \tag{21}$$

Fig. 6 compares empirical estimates of $E\big(\cos(2\phi_i)\big)$ from a simulation of the 1-bit algorithm with (21), and also with similar expressions corresponding to other assumptions on the distribution of ϕ_i e.g. Gaussian, Laplacian and uniform distributions. The plots in Fig. 6 provide more evidence that the empirical distribution of ϕ_i is indeed close to the "exp-cosine" distribution of (20).

Using (21) to evaluate the variances in (17) and (18) allows us to compute $y[n]$ for all n, by using (9) to evaluate the conditional expectation in (7). This completes our analytical model for the average convergence rate of $\mathcal{Y}_{best}[n]$.

The analytical model outlined above can be used to establish some attractive properties of the 1-bit feedback algorithm[6]. We now summarize some of these properties:

1) **Asymptotic Convergence.** Given some mild conditions on the distribution $f_\delta(\delta_i)$, the algorithm converges to perfect coherence *almost surely*, for any initial value of the received phase angles Φ_i.
2) **Monotonicity.** The expected received signal strength at any number of timeslots is a non-decreasing function of N, the number of transmitters, i.e. more transmitters always provide a larger expected received signal amplitude.
3) **Scalability.** The number of timeslots required to converge to within a given fraction e.g. $f = 90\%$ of perfect convergence increases with the number of transmitters, N, but no faster than linearly with N.

The above description and analytical model apply to the static case, where the complex channel gains $h_i = a_i e^{j\psi_i}$, and the phase offsets γ_i are constant with time. In practice, the h_i could change due to Doppler effects in the wireless channel, and γ_i could be affected by oscillator phase noise. If the phase noise is small enough, and the cumulative Doppler variations are small through a significant number of timeslots, we can reasonably model the channel as static.

For the more general time-varying channel, we present a slightly modified version of the feedback algorithm, that allows the transmitters to dynamically track the channel variations. Most of the analytical results for the static case, can be extended to the time-varying case, and this is addressed in detail in Section IV.

III. Experimental Prototype for Distributed Beamforming

We now describe an experimental prototype developed to investigate the performance of distributed beamforming in a practical situation. Figs. 7 and 8 show the block diagammatic representations of the transmitters and BS respectively.

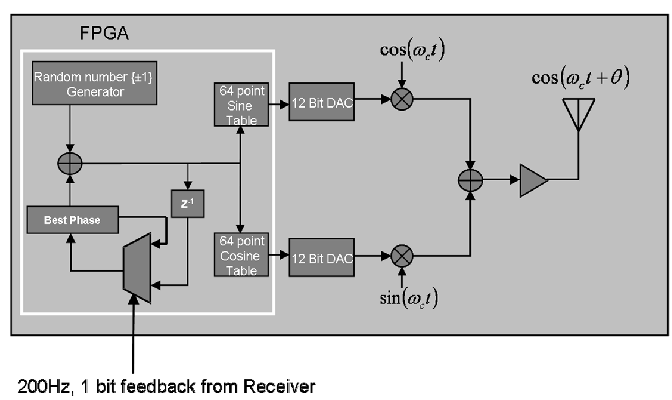

Fig. 7. Block diagram of transmitter prototype

We now describe the operation of the prototypes for the beamformer nodes i.e. transmitters, and the receiver.

A. Description of the prototype design

The Beamformer node consists of an FPGA (Spartan-3 family from Xilinx [12]), two digital to analog converters (DACs), a quadrature modulator, power amplifier and antenna. The baseband processing is done in the FPGA. To implement the random number generator, a simple 1024

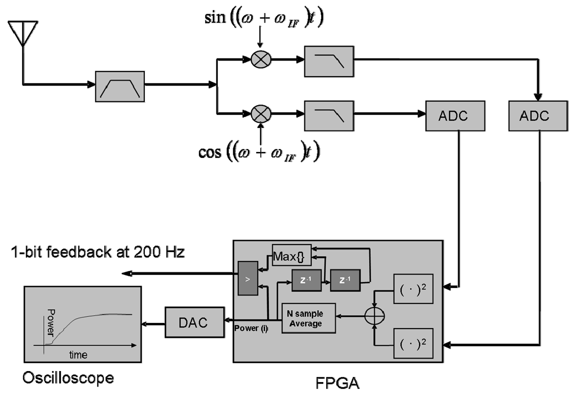

Fig. 8. Block diagram of receiver (BS) prototype

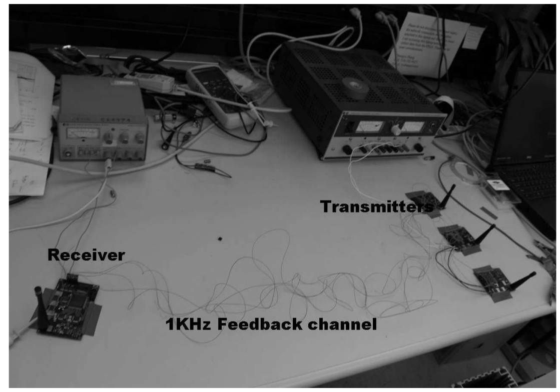

Fig. 9. Photograph of experimental setup

point lookup table was created. The table contains pseudo random numbers ± 1 generated in Matlab. Each beamformer node is loaded with a different table generated in Matlab using a different seed for the random number generator. The phase shifts steps is set to $5.6°$ by using a 64 point sine and cosine table. The lookup address to the table is set to the sum of the output of the random number generator and the address corresponding to the best phase from the previous iteration. When the SNR feedback arrives from the receiver, the best phase is updated if the SNR improved from the previous iteration. We designated one beamformer to be the master device. The master device distributes a low rate clock to the other beamformer nodes so that all nodes can frequency-multiply this clock to the RF frequency, and thus be frequency locked. The clock distribution was done over a simple wired interface but could easily be done wirelessly as well.

The receiver consists of an antenna, bandpass filter, quadrature demodulator, two analog to digital converters (ADCs), an FPGA and a DAC for debugging purposes. An intermediate frequency of 20 kHz was arbitrarily chosen to avoid the problems associated with converting the incoming signal directly to DC. A simple power detector was implemented in the FPGA. The power measurements are averaged over the feedback interval. The receiver feeds back a '1' to the beamformer nodes if the received power is greater than the averaged power measurements from the previous M iterations. M was arbitrarily set to 4 for the experiments. When M was set to 1, the algorithm still converged to large received power, but we observed more oscillatory behavior as the algorithm was trying to continue to search for better phase values even after convergence.

B. Experimental Results

The test setup is shown in Fig. 9. The test consisted of mounting $N = 3$ beamformer nodes across from a receiver node. To test the beamforming gains we initially did not modulate any data onto the carrier. For calibration, each beamformer was turned on while the other 2 beamformer nodes were turned off. We then measured the received power due to each of the beamformer nodes. We then turned on all of the beamformer nodes and ran the algorithm. After

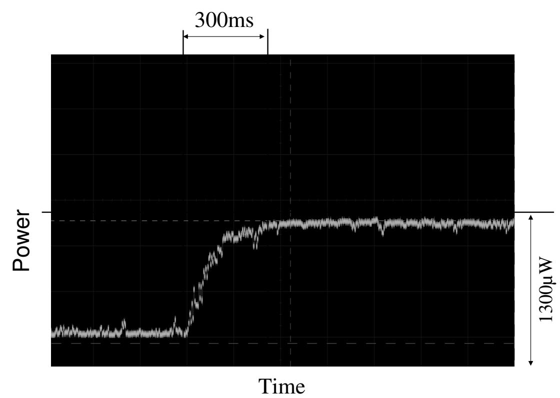

Fig. 10. Received Power Measurement over time

convergence we measured the received power and compared this with the theoretical value we were expecting. This allowed us to measure the beamforming gain. Table I shows the results for one case.

If the beamforming was ideal, the theoretical received power would have been $1370\mu W$ while the actual received power was $1230\mu W$. Thus we were within 90 % of the theoretical limit. We also ran the algorithm while the beamformer nodes BPSK modulated the carrier with a known 10 kbps sequence. The modulation did not affect the convergence time or the beamforming gain as expected. We ran several trials and obtained similar results. The convergence of the algorithm took approximately 60 iterations. With a 200 Hz feedback rate that corresponded to approximately 300 ms. A typical realization of the received power while the algorithm ran is shown in Fig. 10.

Transmitters ON	Received Power (μW)
1	120
2	85
3	280
1, 2 and 3	1230

TABLE I

RECEIVED POWER MEASUREMENTS

IV. Tracking a Time-Varying Channel

Section II-B presents an analytical model for the convergence of the synchronization when the wireless channels from each transmitter to the receiver is static i.e. constant in time. For such channels, the 1-bit algorithm can be shown to converge asymptotically to perfect coherence with *probability 1*. Once converged, the transmitters can use the optimal value $\theta_{best,i}$ obtained from the algorithm to maintain coherent transmission in subsequent timeslots. However in practical cases, the channel phase responses change in time e.g. due to Doppler effects from moving scatterers. For such channels, the channel variations cause the transmitted signals to lose coherence over time: even when the transmitters use the same phase rotation $\theta_{best,i}$, the received phase $\Phi_{best,i}[n] = \gamma_i + \theta_{best,i}[n] + \psi_i[n]$ will not remain the same, because of the change in the channel phase response $\psi_i[n]$. As a result, the received signal strength $\mathcal{Y}_{best}[n] = \left|\sum_{i=1}^{N} a_i e^{j\Phi_{best,i}[n]}\right|$ decreases on average. Fortunately, the 1-bit algorithm can be easily adapted to dynamically adjust the transmitted phase $\theta_{best,i}[n]$. We now present this modified algorithm.

1) At each timeslot n, each transmitter keeps a record $\theta_{best,i}[n]$ of the best known value of its phase rotation, and the receiver keeps an estimate $\mathcal{Z}_{best}[n]$ of the best achievable RSS. Unlike the static case, $\mathcal{Z}_{best}[n]$ is only an estimate of the best achievable RSS $\mathcal{Y}_{best}[n]$ that changes randomly because of channel variations.
2) At timeslot $n+1$, each transmitter generates a random phase perturbation δ_i from some probability distribution $f_\delta(\delta_i)$, and transmits its message signal with an incremental phase rotation δ_i: $\theta_i[n+1] = \theta_{best,i}[n] + \delta_i$. This results in the received phase:

$$\Phi_i[n+1] = \Phi_{best,i}[n] + \delta_i + \Delta_i[n] \tag{22}$$

where $\Phi_{best,i}[n] \doteq \gamma_i + \theta_{best,i}[n] + \psi_i[n]$ and $\Delta_i[n]$ is the channel drift i.e. $\psi_i[n+1] = \psi_i[n] + \Delta_i[n]$.
3) The BS measures the received signal strength, $\mathcal{Y}[n+1] = \left|\sum_{i=1}^{N} a_i e^{j\Phi_i[n+1]}\right|$ and generates a single bit of feedback that is set to '1' if the received signal strength in the current timeslot is better than the estimated best RSS $\mathcal{Z}_{best}[n]$, and '0' otherwise. The BS then broadcasts this bit of feedback to all transmitters.
4) If the feedback bit is '1', the BS updates its value of $\mathcal{Z}_{best}[n+1]$ with the new measured RSS, and the transmitters update the phase rotations $\theta_{best,i}[n+1]$ to retain the perturbations δ_i; otherwise the BS *discounts* its estimated best RSS $\mathcal{Z}_{best}[n]$ by a factor $\rho < 1$ to reflect the expected deterioration due to channel variation, and the transmitters discard the perturbations δ_i.
5) The process is repeated in the next timeslot.

The received phases change due to both the update process and the channel drifts. The update process can be written mathematically as:

$$\mathcal{Z}_{best}[n+1] = \begin{cases} \mathcal{Y}[n+1], & \mathcal{Y}[n+1] > \mathcal{Z}_{best}[n] \\ \rho\ \mathcal{Z}_{best}[n], & \text{otherwise.} \end{cases} \tag{23}$$

$$\theta_{best,i}[n+1] = \begin{cases} \theta_{best,i}[n] + \delta_i[n], & \mathcal{Y}[n+1] > \mathcal{Z}_{best}[n] \\ \theta_{best,i}[n], & \text{otherwise.} \end{cases}$$

$$\Phi_{best,i}[n+1] = \begin{cases} \Phi_{best,i}[n] + \Delta_i[n] + \delta_i[n], & \\ \qquad\qquad \mathcal{Y}[n+1] > \mathcal{Z}_{best}[n] & \\ \Phi_{best,i}[n] + \Delta_i[n], & \text{otherwise.} \end{cases}$$

Unlike the static case, this tracking version of the 1-bit feedback algorithm does not converge to a fixed $\mathcal{Y}_{best}[n]$, but rather to a dynamic steady state. Intuitively, if at any time the received phases $\Phi_i[n]$ become highly coherent, it becomes harder to find "favorable" perturbations δ_i, and therefore, the overall tendency for the RSS is to decrease because of the channel drifts. The steady state balances the tendency of the channel drifts Δ_i to drive the phases away from coherence, and this is partly compensated by the random perturbations δ_i with feedback.

To quantitatively analyze this, we model the drift process $\Delta_i[n]$ as iid across sensors, and stationary and uncorrelated in time with a distribution $f_\Delta(\Delta_i)$. Much of the analysis of Section II-B can now be extended for the time-varying case. In particular, the typicality argument of Section II-C can be used in this case also, and therefore the empirical distribution of the phases $\Phi_i[n]$ at any instant is still given by (20) if the number of transmitters N is large. As before we can write the aggregate effect of the phase perturbations, and channel drift as an increase or decrease in the magnitude $\mathcal{Y}_{best}[n]$ of the received signal, and a rotation of its phase, and we can write an expression similar to (9):

$$\mathcal{Y}[n+1] = \left| C_\delta C_\Delta \mathcal{Y}_{best}[n] + z_1 + jz_2 \right| \tag{24}$$

where $C_\Delta \doteq E\big(\cos\Delta_i\big)$. This also suggests a natural choice for the discounting factor as $\rho = C_\Delta$. This choice would make $\mathcal{Z}_{best}[n] = E\big(\mathcal{Y}_{best}[n]\big)$. As before z_1 and z_2 are uncorrelated, zero mean random variables whose distributions are approximately Gaussian because of the Central Limit Theorem, and their variances can be shown to be respectively:

$$\sigma_{11}^2 = \frac{\alpha N}{2}\Big(\big(1 - C_\delta^2 C_\Delta^2\big) - \big(C_\delta^2 C_\Delta^2 - C_{2\delta} C_{2\Delta}\big) E\big(\cos(2\Phi_{best,i})\big)\Big) \tag{25}$$

$$\sigma_{22}^2 = \frac{\alpha N}{2}\Big(\big(1 - C_\delta^2 C_\Delta^2\big) + \big(C_\delta^2 C_\Delta^2 - C_{2\delta} C_{2\Delta}\big) E\big(\cos(2\Phi_{best,i})\big)\Big) \tag{26}$$

where $C_{2\Delta} \doteq E\big(\cos(2\Delta_i)\big)$, and $\phi_i \doteq \Phi_{best,i} - \Phi_0$, Φ_0 defined as in (12). From (23) and (24), we observe that $\mathcal{Y}_{best}[n]$ is a Markov process, and its transition probability function is defined by:

$$f_M\big(y_2 | y_1\big) \doteq f\big(\mathcal{Y}_{best}[n+1] = y_2 | \mathcal{Y}_{best}[n] = y_1\big) \tag{27}$$

The transition probability function $f_M(y_2|y_1)$ can be expressed in terms of the known Gaussian densities of z_1 and z_2. From $f_M(y_2|y_1)$, we can calculate the steady-state probability density $f_{ss}(y)$ of the Markov chain as the solution to the eigenvalue problem:

$$f_{ss}(y) = \int_{y_1=-\infty}^{\infty} f_M(y|y_1) f_{ss}(y_1) dy_1 \qquad (28)$$

Fig. 11 compares the steady-state distribution $f_{ss}(y)$ computed by solving (28), with a histogram of $\mathcal{Y}_{best}[n]$ obtained from a simulation of the 1-bit algorithm with channel time-variations (after discarding the initial "transient" samples). The excellent agreement between $f_{ss}(y)$ and the histogram shows that the analytical model accurately predicts the behaviour of the algorithm.

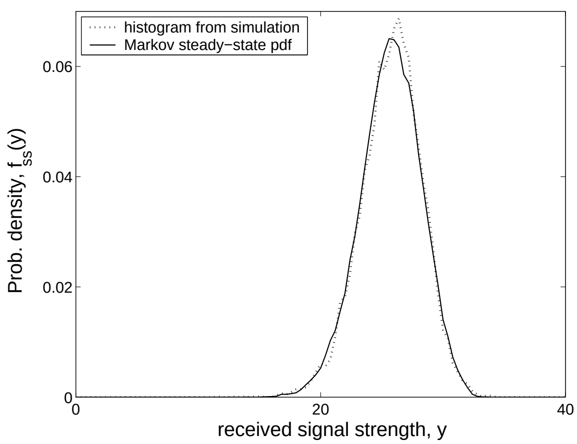

Fig. 11. Distribution of received signal: $N = 40$, channel drift $f_\Delta(\Delta_i) \sim$ uniform$[-\frac{\pi}{25}, \frac{\pi}{25}]$, phase perturbations $f_\delta(\delta_i) \sim$ uniform$[-\frac{\pi}{7}, \frac{\pi}{7}]$

The above analysis of the tracking algorithm as a Markov process can be used to choose the distribution $f_\delta(\delta_i)$ optimally to maximize the average steady state RSS. This analysis is still an ongoing research, but we can make a few general remarks.

1) In order to get good tracking performance, the perturbations δ_i need to be at least as large as the channel drifts Δ_i on average.
2) The perturbations δ_i should not be too large on average, to avoid large fluctuations in the RSS.
3) The effect of phase jitter (i.e. fluctuations in the phase γ_i) is similar to channel variations. If we apply the above steady state analysis to the experimental results of Section III, the steady state beamforming gain correspond to a rms phase jitter of about 10°. This is consistent with a visual observation of the RF carrier signals on an oscilloscope.

V. Conclusions

The results presented in the previous sections are promising and show that large SNR gains are achievable under practical conditions using distributed beamforming. This initial investigation leaves substantial scope for future work. The most important open issue is measuring and optimizing the algorithm for tracking time-varying channels. Our experiments in a static laboratory environment does not provide information about the tracking performance of the algorithm. Our analytical model is intuitive and provides valuable insights; it is conceptually possible to use the analytical model and solve for the optimal choices of several important parameters including the distribution $f_\delta(\delta_i)$ and the discounting factor ρ. However we do not yet have closed-form solutions for these important parameters, and characterizing these parameters compactly is an open problem for future work.

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