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# Control of a three-coleader formation in the plane

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**Abstract**

This paper considers a formation of three point agents moving in the plane, where the agents have a cyclic ordering with each one required to maintain a nominated distance from its neighbour; further, each agent is allowed to determine its movement strategy using local knowledge only of the direction of its neighbour, and the current and desired distance from its neighbour. The motion of the formation is studied when distances are initially incorrect. A convergence result is established, to the effect that provided agents never become collinear, the correct distances will be approached exponentially fast, and the formation as a whole will rotate by a finite angle and translate by a finite distance.


*Keywords:* Decentralized control; Multiple agent control; Formation control

## 1. Introduction

In recent years, there has been an increasing number of contributions dealing with the control of agent formations. Among the older contributions, we note e.g. [1,9–12,15–17]. Roughly speaking, a set of agents is prescribed, to move in two or three-dimensional space, possibly but not necessarily point agents, and it is envisaged that they will move as a formation from point A to point B, possibly executing some mission, possibly avoiding obstacles, etc. The words ‘move as a formation’ have the usual meaning of everyday language: the formation at one instant of time is congruent to the formation at another instant of time, or equivalently, all inter-agent distances are preserved over all time. Many of the cited early contributions deal with the question of just what inter-agent distances or other constraints, or indeed how many, are needed to ensure such motion.

Exactly how motion is achieved in a stable way is an issue of great interest, and recent papers have tended to focus more on the control laws required, [5–8,13,14]. It has been observed that if some inter-agent distances are preserved, for example $2n-3$, where $n$ is the number of agents in a two-dimensional formation of point agents, then all inter-agent distances can be consequentially preserved, and a scalable and even distributed control algorithm can be envisaged. Other schemes for control of formation shape can be envisaged too; for example, some angles can be preserved, in addition to distances, and as an alternative to some distances.

In this paper, like many predecessors, we consider control of formation shape based on inter-agent distance preservation. What distinguishes this work, however, from most but not all work to this point is that we formulate the task of controlling the distance between two agents to a set-point value as a directed one, by assigning it to only one of the two agents. We consider a particularly simple formation, one with just three agents, and they are in a cyclic relation to each other, i.e. agent 1 should maintain a distance from agent 2, agent 2 should maintain a distance from agent 3, and agent 3 should maintain a distance from agent 1.

Among works dealing with what one might term *directed* formation control, we note that of [1,5,10,15]. Directed formation control is straightforward if the underlying directed graph depicting the control structure is acyclic; there is an induced partial ordering of the agents, and in control terms, the system equations are triangularly coupled. This is because follower agents are influenced by leader agents, but cannot influence the leader agents due to the acyclicity of the graph. Challenging

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problems therefore arise when the graph has cycles, and indeed Tabuada et al. [15] emphasize cyclic graphs, while maintaining a great degree of generality about the nature of the constraints linking the agents. Baillieul and Suri [1] raise the possibility of considering cyclic structures where there are distance measurements used to achieve control, and argue that such structures are inherently flawed, at least in the presence of noise/bias errors etc. Lee and Spong [5] consider directed structures, but with the requirement that the underlying graph be balanced (i.e. each node has the same number of inwardly and outwardly directed edges, though a variation is possible with a concept called weighted balancing), and in fact their work is aimed at a different problem (flocking) rather than preservation of the shape of a two-dimensional formation. Nevertheless, preliminary work of the authors confirms the notion that balanced graphs will also allow efficacious treatment of distance-based formation shape preserving problems. Indeed, this paper is considering virtually the simplest two-dimensional formation with a simple balanced graph. If the underlying graph is balanced in more general formations than the one considered in this paper, this preliminary work suggests that it could be relatively straightforward to construct a control law. For a nonbalanced structure, a more sophisticated procedure is needed, and is the subject of other work by the co-authors.

Earlier on in this work, it was identified that the concept of *graph rigidity* could helpfully underpin much of the control law development. A necessary condition for a distributed control law to exist which will stabilize a formation is that (in the undirected graph case where two agents work together to maintain the correct separation) the underlying graph is rigid, a point specially emphasized in the contributions of Olfati-Saber and colleagues, e.g. [9–12]. In the directed graph case, rigidity is not enough. One needs a further concept, termed *persistency*, see [4,19]. This includes rigidity, but overlays this with a further condition that rules out certain information-flow or sensing patterns, (corresponding to particular choices of one member of an agent pair to control the distance between that pair) that are otherwise consistent with the rigidity property. In a persistent graph, it remains possible to have cycles.

The purpose of this work is to demonstrate in detail how control based on distance preservation can be achieved when a cycle is present. We also comment on the difficulties raised by Baillieul and Suri [1].

In the next section we formulate the main problem, while the third section states the control law. The main stability analysis is performed in Section 4; it turns out that there are four possible equilibria for the closed-loop equations, three of which are unstable (and incidentally pathological) and do not correspond to the desired formation shape. The remainder of the paper discusses briefly the effect of noise and bias, and offers concluding remarks.

## 2. Problem description

We consider a formation comprising three coleaders, each with one degree of freedom. The agents are point agents, massless, and holonomic. The three agents are initially at incorrect distances from one another. Our aim is to set up the equations with a control law for restoring the correct distances and prove a form of stability result.

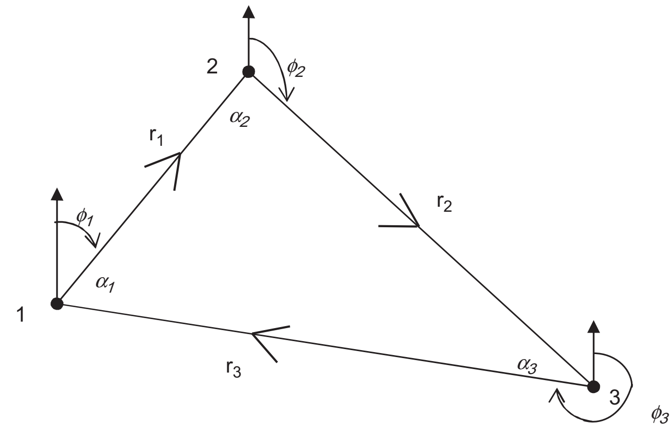


Fig. 1. Formation consisting of three coleaders.

Notation is defined in reference to Fig. 1, and is as follows: $\phi_1$ is the angle between north and the direction of agent 2, as seen from agent 1, with $\phi_2$ and $\phi_3$ being defined analogously; $r_i$ is the current distance from agent $i$ to agent $i+1$ (agent 1 being identified with agent 4, here and subsequently); $d_i$ is the distance which ought to be maintained between agent $i$ and $i+1$ (the 'correct' distance); and $\alpha_i$ is the internal angle of the triangle formed by the three agents, at agent $i$.

Agent $i$ knows $d_i$, $r_i$, and the direction of agent $i+1$. Agent $i$'s control law can only use this information.

We shall make two standing assumptions. First, it is assumed that for $i \neq j \neq k$, the triangle inequality $d_i + d_j > d_k$ holds; thus the steady state to which the formation is supposed to tend is well-defined as a triangle. Second, it is assumed that at no point in the motion, do any of the $r_i$ become zero; nor does $r_i$ tend to zero as time tends to infinity. Equivalently, no two agents ever coincide in their position. We shall later indicate a sufficient condition on the initial conditions that ensures this requirement is met. (A more sophisticated control law than that introduced in the next section, which introduces a repulsive force when agents become too close, will achieve the same effect.) A consequence of the assumption is that the angles $\alpha_i$ are well defined throughout the motion, even if the three agents become collinear.

## 3. Control law

The movement rule for agent $i$ is: to move in the *direction* of agent $i+1$, and with speed $s_i$ defined by the following equation:

$$s_i = -(d_i - r_i). \tag{1}$$

Evidently, if $r_i < d_i$, so that agent 1 is too close to agent 2, this equation assigns a negative speed, so that agent 1 then moves away from agent 2, along the line joining agents 1 and 2.

This means that we have $\dot{x}_i = s_i \sin \phi_i$ and $\dot{y}_i = s_i \cos \phi_i$, where $(x_i, y_i)$ are the coordinates of agent $i$.

We remark that at any instant of time, the angles $\alpha_i$ are standard functions of the $r_j$:

$$\alpha_i = f(r_{i-1}, r_i, r_{i+1}) = \cos^{-1}\frac{r_{i-1}^2 + r_i^2 - r_{i+1}^2}{2r_{i-1}r_i}. \tag{2}$$

Since we have precluded the possibility of allowing any $r_i$ to be zero, it is evident that the three $\cos\alpha_i$ are well defined in terms of the $r_i$.

Note also that a full description of what is happening at agent 1 includes not just the nonzero $\dot{r}_i$ change but also a 'kinematically induced' $\phi_1$ change, arising because agent 2 is moving. Thus the direction of movement of agent 1 changes, in general all the time. The same is of course true of agents 2 and 3. This suggests that in addition to the differential equations in which $\dot{r}_i$ appears, we need differential equations in which $\dot{\phi}_i$ appears.

## 4. Differential equation model of the closed-loop system

In the immediately following material, we set up a full set of differential equations to describe the formation using the control law indicated above. To begin with we shall show that $\phi_2$ and $\phi_3$ can be expressed as functions of $\phi_1$ and the $r_i$. So we focus on the variables $r_i$ and $\phi_1$. Indeed, we shall first find a coupled set of differential equations for the three $r_i$; then we shall find a differential equation which expresses $\dot{\phi}_1$ as a function of just the $r_i$.

### *4.1. Dependence of $\phi_2$ and $\phi_3$*

It is clear from Fig. 1 that

$$\phi_2 = \phi_1 + (\pi - \alpha_2) \tag{3}$$

and

$$\phi_3 = \phi_2 + (\pi - \alpha_3), \tag{4}$$

while also the following equation holds but is not independent, because the $\alpha_i$ sum to $\pi$:

$$\phi_1 = \phi_3 + (-\pi - \alpha_1). \tag{5}$$

(Actually, there will be other choices of triangle orientation and angle size for which equations such as (3) are only correct modulo $2\pi$.) Now recall that the $\alpha_i$ are expressible in terms of the $r_i$; it follows easily that $\phi_2$ and $\phi_3$ are expressible as functions of $\phi_1$ and the $r_i$.

We can view the fact that such relations should exist in another way. Evidently, because $(x_2 - x_1) + (x_3 - x_2) + (x_1 - x_3) = 0$ with a similar equation for the $y_i$, we have $r_1 \sin\phi_1 + r_2 \sin\phi_2 + r_3 \sin\phi_3 = 0$ and $r_1 \cos\phi_1 + r_2 \cos\phi_2 + r_3 \cos\phi_3 = 0$ and when the $r_i$ and $\phi_1$ are known, we have two equations for the two unknowns $\phi_2$ and $\phi_3$.

### *4.2. Coupled differential equations for the $r_i$*

Observe that the distance between agent 1 and agent 2 changes as a result of the movement of both agents. The movement of agent 1 has magnitude $-(d_1 - r_1)$ in the direction from agent 1 to agent 2, and thus contributes $(d_1 - r_1)$ to $\dot{r}_1$. The movement of agent 2 has magnitude $-(d_2 - r_2)$ in the direction of agent 3, and consequently the component in the direction of the line joining agent 1 to agent 2 is $(d_2 - r_2)\cos\alpha_2$. Combining these two movements, we see that

$$\dot{r}_1 = (d_1 - r_1) + (d_2 - r_2)\cos\alpha_2.$$

Defining the error variables $e_i = r_i - d_i$, we see that

$$\begin{bmatrix} \dot{e}_1 \\ \dot{e}_2 \\ \dot{e}_3 \end{bmatrix} = \begin{bmatrix} -1 & -\cos\alpha_2 & 0 \\ 0 & -1 & -\cos\alpha_3 \\ -\cos\alpha_1 & 0 & -1 \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix}. \tag{6}$$

Note that this equation is *not* a linear equation since the $\alpha_i$ are functions of the $r_i$, or equivalently of the $e_i$, given the values of the $d_i$. For example, $\cos\alpha_2 = [(e_1 + d_1)^2 + (e_2 + d_2)^2 - (e_3 + d_3)^2]/2(e_1 + d_1)(e_2 + d_2)$. Nevertheless, it is straightforward to prove solution existence and asymptotic stability.

**Theorem 1.** *Consider the equation set* (6). *Then under the standing assumptions and in particular for all initial conditions such that no $r_i$ is ever zero along the solution trajectory, solutions are well defined on the semi-infinite time interval. Moreover, if for some positive $\varepsilon$, it is guaranteed that $\alpha_i \in [\varepsilon, \pi - \varepsilon]$ for $i = 1, 2, 3$, then the equation set is globally exponentially convergent.*

**Proof of Theorem 1.** Note that the assumption that no $r_i$ is ever zero ensures that the $\cos\alpha_i$ are well defined on every motion and local solution existence is guaranteed. Write the equation set as $\dot{e} = F(e)e$, where $e = [e_1\ e_2\ e_3]^{\mathrm{T}}$. Adopt as a Lyapunov function $V(e) = e^{\mathrm{T}}e$. Now a straightforward calculation establishes that $F(e) + F(e)^{\mathrm{T}}$ is negative semi-definite, so that $\dot{V} \leqslant 0$; hence the $e_i$ are bounded and the possibility of a finite escape time is ruled out, and solution existence on the semi-infinite time interval follows. Under the additional assumption that $\alpha_i \in [\varepsilon, \pi - \varepsilon]$ for $i = 1, 2, 3$, it is immediate that $F(e) + F(e)^{\mathrm{T}}$ is uniformly negative definite under the constraint on the $\alpha_i$; indeed the negative of the matrix is diagonally dominant given the constraints on the $\alpha_i$. Accordingly, the Lyapunov function $V(e) = e^{\mathrm{T}}e$ obeys $\dot{V} < -\gamma V$ for some positive $\gamma$ and the exponential convergence property follows.

**Remark 2.** The condition on the $\alpha_i$ in the second part of the theorem hypothesis requires that the triangle defined by the three agents be bounded away from becoming a straight line.

**Remark 3.** If at time zero, there holds $e_1^2(0) + e_2^2(0) + e_3^2(0) < \min[d_1^2, d_2^2, d_3^2]$, the decreasing nature of $V(e)$ means that at no subsequent point on the motion can one have $e_i^2 = d_i^2$ for some $i$, and therefore no $r_i$ can ever become zero. The set of initial conditions for which $r_i$ will never become zero is evidently sizeable, and larger than that provided by the sufficient condition just stated.

We will now consider in more detail the behaviour of (6) in the absence of the condition on the $\alpha_i$ in the second part of the theorem, which precludes collinearity arising in a motion. Of course, there remains in force the standing assumption excluding the possibility that $r_i$ can be zero.

First, we consider the existence of other equilibrium points than $e=0$. Any equilibrium point, call it $e_0$, must have the property that $F(e_0)e_0=0$. A necessary property is that $F(e_0)$ is singular: now since $F(e_0)$ is in the first instance determined by the values of the $\alpha_i$ it is straightforward to see that since the $\alpha_i$ are nonnegative and sum to $\pi$, $F(e_0)$ is singular if and only if one $\alpha_i$ is $\pi$ and the other two are zero. Without loss of generality suppose that $\alpha_2$ is $\pi$.

Under this constraint, the matrix $F(e_0)$ is easily checked to have a kernel of dimension 1, viz. the set of all vectors $e$ satisfying $e_1=e_2=-e_3$. Observe however also that the condition $\alpha_2=\pi$ forces the following equality among the sides of the triangle: $r_1+r_2=r_3$. Consequently, $e_1+e_2-e_3=-(d_1+d_2-d_3)$. Putting these constraints on the $e_i$ together, we see then that there is a single equilibrium point associated with the condition that $\alpha_2=\pi$, viz., $e_1=e_2=-e_3=-(d_1+d_2-d_3)/3$. Thus we have proved:

**Corollary 4.** *Consider the equation set (6). The only equilibrium points are*

$$[0\ 0\ 0]^{\mathrm{T}}, \quad k_1[-1\ \ -1\ \ 1]^{\mathrm{T}},$$
$$k_2[1\ \ -1\ \ -1]^{\mathrm{T}}, \quad k_3[-1\ 1\ \ -1]^{\mathrm{T}}$$

*where* $k_1=(d_1+d_2-d_3)/3$, $k_2=(d_2+d_3-d_1)/3$, $k_3=(d_3+d_1-d_2)/3$, *and corresponding, respectively, to the three agent collinearity possibilities* $\alpha_2=\pi$, $\alpha_3=\pi$, $\alpha_1=\pi$.

**Remark 5.** It is not hard to verify that none of these equilibrium points is consistent with any $r_i$ taking the value zero, i.e. any $e_i$ taking the value $-d_i$. Indeed, one can check that were $e_i=-d_i$, the triangle inequality condition satisfied by the $d_j$ would be violated.

Next, consider any (equilibrium or nonequilibrium) point for which $\alpha_2=\pi$; at such a point, there necessarily holds $r_1+r_2=r_3$ and consequently, $e_1+e_2-e_3=-(d_1+d_2-d_3)$. Now use (6) with the specialized values of the $\alpha_i$ to verify that $\mathrm{d}/\mathrm{d}t(e_1+e_2-e_3)=0$. In turn, this implies $\mathrm{d}/\mathrm{d}t(r_1+r_2-r_3)=0$. Thus we have proved that if the agents are collinear at one point on the system trajectory, they will stay collinear after that, i.e. the collinearity conditions define invariant manifolds; we shall also prove that the equilibrium points identified above are, when motions are a priori restricted to lying within the identified manifold, stable:

**Corollary 6.** *Consider the equation set* (6). *Then there exist three invariant motions of the system trajectory defined by* $e_1+e_2-e_3=-(d_1+d_2-d_3)$ *corresponding to the collinearity condition* $r_1(t)+r_2(t)-r_3(t)=0$ (*and two similar alternative equations corresponding to two other collinear orderings of the three agents*). *Such motions approach the equilibirium points identified in Corollary* 4.

**Proof of Corollary 6.** It remains to establish the stability part of the result. Along the trajectory defined by $e_1+e_2-e_3=-(d_1+d_2-d_3)$, we have $\alpha_1=0$, $\alpha_2=\pi$, $\alpha_3=0$ and so $F(e)$ is constant. There is one zero eigenvalue (corresponding to the fact that $e_1+e_2-e_3$ is constant), and the other two are easily checked to be stable. In fact, one can verify that

$$\begin{bmatrix} \dot{e}_1-\dot{e}_2 \\ \dot{e}_2+\dot{e}_3 \end{bmatrix} = \begin{bmatrix} -1 & 1 \\ -1 & -2 \end{bmatrix} \begin{bmatrix} e_1-e_2 \\ e_2+e_3 \end{bmatrix}.$$

Thus trajectories on $r_1+r_2=r_3$ approach the equilibrium point $e_1=e_2=-e_3$. □

### 4.3. A differential equation for $\phi_1$

The velocity of agent 2 has components $(\dot{x}_2, \dot{y}_2)$. Evidently (see Fig. 1) we can consider the component of motion of agent 2 in a direction at right angles to the line joining agent 1 to agent 2 and obtain

$$r_1\dot{\phi}_1=-(d_2-r_2)\sin\alpha_2, \tag{7}$$

so that

$$\dot{\phi}_1=r_1^{-1}(r_2-d_2)\sin[f(r_1,r_2,r_3)]. \tag{8}$$

Note that $\phi_1$ does not appear on the right side of this equation. Now recall our standing assumption that the initial values of $r_i$ for the formation preclude the possibility that the subsequent motion ever allows $r_1=0$; as we know, this will be so certainly if the initial values are within a prescribed distance of the desired values $d_i$. Then $\phi_1$ will be well-defined for all time, and the integral of the equation will be well defined and provide the change in $\phi_1$ from the initial position to the final position; evidently and not surprisingly, this change is independent of the initial orientation of the formation.

Now let us return to the examination of the extra equilibrium points identified in the previous subsection. Consider that associated with $\alpha_2=\pi$. From (3), we see that

$$\begin{aligned}\dot{\alpha}_2&=\dot{\phi}_1-\dot{\phi}_2\\&=r_1^{-1}(r_2-d_2)\sin\alpha_2-r_2^{-1}(r_3-d_3)\sin\alpha_3.\end{aligned}$$

In the vicinity of the equilibrium point $e_1=e_2=-e_3=-\frac{1}{3}(d_1+d_2-d_3)$, corresponding to $\alpha_2=\pi$, it is evident that $\dot{\alpha}_2$ is negative. This means that the equilibrium point is unstable (in fact, given the result of Corollary 6, we see that it is a saddle point). On the other hand, consider a point such that $e_1+e_2-e_3=-(d_1+d_2-d_3)+\varepsilon$ for some small positive $\varepsilon$. Such a point corresponds to having $\alpha_2$ close to but less than $\pi$. Suppose that also, $e_2$ is positive and $e_3$ is zero. Then $\dot{\alpha}_2$ will be positive. This suggests that some parts of the manifold $r_1+r_2-r_3=0$ may be attractive.

### 4.4. The overall system

The differential equations describing the overall system are (taking into account that $e_i=r_i-d_i$) (6) and (8), and the other variables are given by (2)–(4).

It is self-evident that this differential equation system is triangularly coupled, and that the $r_i$ converge exponentially fast to $d_i$ provided we agree to exclude motions on the invariant sets identified in Corollary 6. Further, though the differential equation for $\phi_1$ is not asymptotically stable, it is evident that this variable will converge since the right-hand side of the equation decays exponentially fast to zero. The convergence of $\phi_2$ and $\phi_3$ is self-evident. Since

$$\dot{x}_i = s_i \sin \phi_i = -(d_i - r_i) \sin \phi_i$$

and $(d_i - r_i)$ converges exponentially fast, we see that each $x_i$ also converges to a fixed value, and similarly for the $y_i$.

The change between $t = 0$ and $\infty$ of $\phi_1, x_1$, and $y_1$ defines the total move of the formation as a whole (with reference to one vertex of it, and one direction of an agent pair) while the formation is correcting its inter-agent lengths. Note that agent 1 is *not* the leader of the formation, and the conclusion could have been described using $\phi_2, x_2, y_2$ or $\phi_3, x_3, y_3$.

## 5. Effects of noise and bias

One can expect that there will be errors in measuring the value of key variables, either due to noise or bias. Because the error equation (6) has an exponential stability property, the effect on the inter-agent distances will simply be that the $e_i$ will converge to some neighbourhood of 0. However, the variables $\phi_i, x_i$, and $y_i$ will in general not converge to some fixed point. Rather, they will continue or wander, or in the case of a systematic bias, the formation may end up rotating and/or translating with constant angular or linear velocity.

In practice, either there will be an additional external input to the system equations reflecting for example a requirement for the formation to move as a whole from one region of the plane to another (and provided this is posed in closed-loop form, the otherwise drifting formation modes will almost certainly be stabilized.) Alternatively, deadzones may be introduced to govern the motion of the individual agents; the formation shape will then not exactly converge, but if the deadzone is bigger than the errors and biases, these will not affect the motion any longer. Recent work [3] of Fidan and one of the coauthors has investigated this possibility in some detail for a related one-dimensional formation problem.

## 6. Conclusions

This paper has dealt with a particular example of a *cyclic and minimally persistent* formation, and demonstrated the existence of a control law which will stabilize the shape of the formation. Apart from earlier results covering formations with acyclic graphs, this is perhaps the first result of this type, involving as it does a 3-cycle. (Of course, as indicated in the introduction, there are certainly results dealing with the stabilization of formations in which distance constraints are implemented by two-way control, each agent at the end of a link playing a role in the stabilization or restoration of correct distances).

We adopted a particular form of control law, albeit one that is heuristically motivated. It would seem straightforward to vary the law to make the speed not as given in the paper by (1) but rather as any first-third quadrant smooth and sector bounded function of $(d_i - r_i)$. We also formulated equations of motion in terms of the six position coordinates of the three agents; this results in a nonlinear set of equations which are much harder to analyze, because the linearized system contains three zero modes, and center manifold theory is then needed to draw a conclusion concerning the nonlinear system. The benefit of our analysis is that the coupling between, on the one hand, the $r_i$ or equivalently $e_i$ equations which deal with the formation shape, and on the other hand, the equations dealing with the overall formation position and orientation (as if it were a rigid body), are triangularly coupled.

Separate work [2], so far unpublished, of the authors together with Cao has considered an alternative parametrization for the same problem and with a different law establishes that if the initial position of the agents is not collinear, the correct triangular shape will always be attained. Other separate work [18], also so far unpublished, of some authors has considered minimally persistent formations containing cycles and with more than three agents, and ideas similar to those of this paper can be applied. However, the control laws are not always stabilizing, and it is often necessary that different gains be used by different agents in order to secure stability. In this other work, a matrix associated with the formation termed the *rigidity matrix* plays a major role in defining the control laws.

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