A DISTRIBUTED CONSENSUS APPROACH TO SYNCHRONIZATION OF RF SIGNALS

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ABSTRACT

We propose a consensus-based algorithm for the synchronization of carrier signals in a wireless network. This work is motivated by recent progress on distributed beamforming and other cooperative MIMO techniques that require synchronized RF signals among all the cooperating nodes in a network and is aimed at addressing the limitations of the centralized master-slave approach used in previous work in this area. Our proposed algorithm is based on a variation of the classic Kuramoto model for the synchronization of coupled oscillators and is well-suited for a digital baseband implementation. We describe our proposed algorithm in detail and present initial results that show that this algorithm achieves global frequency lock given only that the network is connected i.e. there exists (possibly multi-hop) paths for every node to transmit and receive a signal from every other node.

***Index Terms*—** synchronization, cooperative communication, consensus, coupled oscillators

1. INTRODUCTION

We propose a consensus-based algorithm for the synchronization of carrier signals in a wireless network. Carrier synchronization is a pre-requisite for cooperative MIMO schemes; under these schemes a number of single-antenna nodes in a wireless networks organize themselves into a virtual antenna array and cooperatively use MIMO techniques to obtain substantial diversity, multiplexing and energy efficiency gains.

Recently such techniques, especially cooperative beamforming [1, 2] have attracted significant interest from researchers and practitioners and their practical feasibility has been demonstrated in several experimental prototypes [3, 4]. The recent work in [4] is especially relevant to our work because it shows that synchronization of high frequency radio frequency signals can be performed using purely digital signal processing techniques, and such techniques can be implemented on commodity wireless platforms such as software-defined radios.

To see the need for carrier synchronization, we begin with the observation that each node in a network derives its RF signal from its own local oscillator. Two oscillators set to the same nominal frequency would in general have a small but non-zero frequency offset between them because of manufacturing tolerances and temperature and other uncontrollable environmental factors. Furthermore all oscillators undergo random drift over time. Frequency offsets between nodes in a virtual array manifest themselves as time-varying channel effects and can substantially negate the gains from the cooperative communication. Thus, it is necessary to explicitly correct for frequency offsets for virtual array applications.

While our work is motivated by the cooperative wireless communication problem, the consensus-based approach to synchronization of high-frequency signals also has a wide range of other interesting applications such as distribution of clock signals in digital ICs and clock recovery in wireline telecommunication systems.

1.1. Background

There is a large literature on applying consensus-based methods to the problem of synchronization in networks. One class of synchronization problems deals with modeling natural phenomena that involve spontaneous synchronzation of a set of distributed nodes. Examples of such phenomena include the flashing of fireflies [5] and circadian biological rhythms [6].

There is also a significant body of recent work on techniques for network time synchronization including methods specifically designed for wireless networks [7]. We note that the time synchronization problem requires a very different approach compared to carrier synchronization [1] because of the high frequency RF signals involved in most wireless communication systems of interest.

The simple, special case of our proposed consensus algorithm for a 2-node network was analyzed in [8]; it was shown for the 2-node case that the algorithm has the remarkable property of global convergence i.e. starting with arbitrarily large initial frequency offsets, the two nodes are guaranteed to achieve frequency lock. This shows how powerful the consensus approach can be; extending the analysis of [8] to the

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generalised algorithm described in this paper is an important topic for future work.

Finally, this paper is also related to the literature on modeling of coupled oscillators; specifically our algorithm can be thought of as a second order variant of the classic Kuramoto model [9]. This connection is explored in more detail in [8].

Outline. The rest of the paper is organized as follows. We describe our formulation of the synchronization problem and introduce our consensus algorithm in Section 2. The consensus approach is contrasted with the master-slave approach in Section 3. Section 4 presents results from numerical simulations to illustrate the performance of the consensus algorithm and Section 5 concludes.

2. DISTRIBUTED CONSENSUS ALGORITHM

We consider a network of N nodes, the network connectivity being represented in a directed graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$, where $i \in \mathcal{V} = 1 \dots N$ is the set of vertices representing the N nodes and $\mathcal{E}$ is the set of edges representing wireless links. Let A_{ij} and θ_{ij} represent the magnitude and phase of the channel gain (assumed frequency non-selective) from node j to node i; of course $A_{ij} = 0$ and θ_{ij} is undefined when $(i, j) \notin \mathcal{E}$. Let $\mathcal{N}_i$ denote the *neighborhood* of node i i.e. $\mathcal{N}_i \equiv \{j : A_{ij} > 0\}$ represents the set of all nodes that have an incoming link to node i.

Let us denote the carrier signal at node i as $c_i(t) = \cos(\phi_i(t))$, where $\phi_i(t)$ is the instantaneous phase of the oscillator at node i with respect to some arbitrary global reference. Thus the instantaneous (angular) frequency ω_i at node i is

$$\omega_i(t) = \frac{d\phi_i(t)}{dt} \qquad (1)$$

Note that the instantaneous frequency measured in cycles per second or Hertz is simply $\frac{\omega_i(t)}{2\pi}$.

Under our consensus algorithm, each node i broadcasts its carrier signal $c_i(t)$ to all of its neighbors. Thus the total received signal $r_i(t)$ at node i is the superposition of the carrier signals from its neighbors after attenuation and phase shifting by the channel:

$$r_i(t) = \sum_{j \in \mathcal{N}_i} A_{ij} \cos(\phi_j(t) + \theta_{ij}) \qquad (2)$$

The goal of the consensus algorithm is to use this received signal at node i to adjust its instantaneous frequency in such a way that eventually leads to all the nodes getting frequency locked.

Remark. Half-duplexing contraints (i.e. the isolation requirements between the transmit and receive sections of each node) prevent each node from receiving its neighbors' transmission of their carrier signal while simultaneously transmitting its own carrier signal in the same frequency band. Thus some kind of multiplexing scheme is necessary to implement this algorithm in real-world networks. The tradeoffs between different types of multiplexing schemes are beyond our scope here; as an example, we can use the frequency multiplexing scheme described in [4].

Our proposed consensus algorithm is a simple control scheme where each node i uses its received signal $r_i(t)$ to adjust its instantaneous frequency as follows:

$$\frac{d\omega_i(t)}{dt} = \beta \sum_{j \in \mathcal{N}_i} \sin\Big(\phi_j(t) - \phi_i(t) + \theta_{ij} + \alpha\omega_i(t)\Big) \qquad (3)$$

In the desired consensus state, we would have $\frac{d\omega_i(t)}{dt} = 0, \forall i \in 1 \dots N$, and each node then has a sinusoidal signal at a constant frequency that is equal at all nodes.

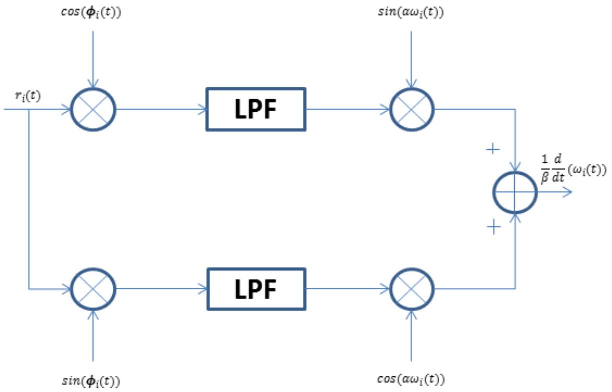

Fig. 1. Block diagram representation of consensus algorithm.

In (3), β and α are parameters that determine the rate of convergence; however because of the highly non-linear kernel in (3), the dependence is not straightforward. Note that the parameter α has the dimensions of time, and roughly speaking it determines the sensitivity of the algorithm. In other words, the algorithm will be relatively insensitive to frequency deviations that are much smaller than $\frac{1}{\alpha}$.

An implementation of this algorithm in block diagram form is shown in Fig. 1. As noted earlier, this algorithm can be thought of as a second-order variation of the Kuramoto model of coupled oscillators. However, the ω_i term on the RHS of (3) does not appear in the Kuramoto model; this term is critically important for the stability of our algorithm.

3. COMPARISON WITH A MASTER-SLAVE ARCHITECTURE

As noted earlier, previous work in this area has invariably used a master-slave architecture for achieving frequency lock between the carrier signals at all nodes; under such an architecture (see Fig. 2 for an illustration), a designated "master" node transmits a common reference signal for "slaves" to lock on to. This master node could be one of the nodes within the network, or it could be an external source such as a GPS satellite.

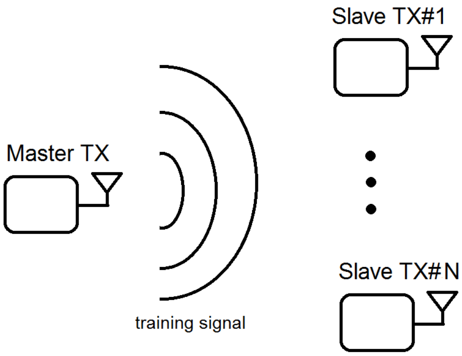

Fig. 2. The traditional Master-Slave architecture for carrier synchronization.

The master-slave architecture has the advantage of simplicity and lack of any need for coordination, it suffers from several inherent limitations.

1. *Requirement of a star-topology.* The master-slave architecture requires that every slave node be able to receive the reference signal from the master node and thus does not directly apply to multi-hop ad-hoc wireless networks. In contrast, our proposed algorithm works for any multi-hop network where all nodes are reachable.

2. *Non-uniform signal to noise ratios.* The residual phase noise in the synchronized signal at each slave is proportional to the noise in the reference signal received from the master node. Thus slave nodes that are far away from the master, or have a weak channel may have a noisy carrier signal after synchronization. In our proposed algorithm, the noise in the reference signal at each node is essentially determined by the signal from its nearest neighbor(s) which can be much stronger than the signal from a central master node.

3. *Stability of the synchronized signal.* Under the master-slave architecture, the slave nodes lock to the master node's oscillator, and thus all the slave nodes will follow the random drifts of the master oscillator. Our proposed consensus-based approach offers the possibility of "averaging out" the uncorrelated drifts of different oscillators and potentially yields a more stable synchronized carrier signal.

4. *Single point of failure.* The master-slave setup also suffers from the limitation of every centralized architecture in having a single point of failure. Thus even a temporary outage of the master node disables the whole network. Our algorithm is completely decentralized and therefore robust to the failure of any subset of participating nodes - as long as the network remains connected.

Thus, the consensus based approach can potentially achieve a synchronized signal that is less noisy, more stable and is more scalable to large networks and robust to link and node failures.

4. RESULTS

We now present some simulation results to demonstrate the working of the consensus algorithm. For our simulations, we consider the $N = 10$ node network with the connectivity graph shown in Fig. 3. For simplicity we used a symmetric graph i.e. $A_{ij} = A_{ji}$, $\forall i, j$. Initially the frequencies are randomly chosen from the range 0 to 10 kHz. This is a typical range for the relative frequency offset of two oscillators with a frequency error of 10 ppm operating in the 1 GHz spectrum. The gains A_{ij} were chosen randomly from a 10 dB range.

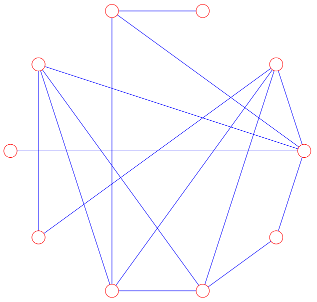

Fig. 3. Connectivity graph for the simulations.

Fig. 4 shows the instantaneous frequency $\frac{1}{2\pi}\frac{d\phi_i(t)}{dt}$ for all the nodes $i \in 1 \dots N$ as a function of time and we can see that the frequencies all converge to around 5.5 kHz within approximately 100 seconds.

5. CONCLUSION

In this paper, we described a novel, distributed consensus-based algorithm for the synchronization of carrier signals among the nodes in an ad-hoc wireless network. We highlighted several advantages of this approach compared to a traditional, centralized master-slave approach and presented

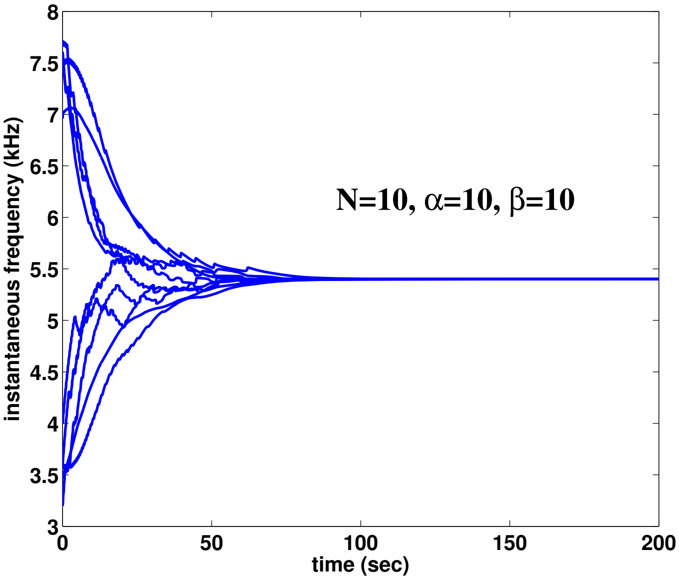

Fig. 4. Frequency locking using the consensus algorithm.

numerical results with realistic parameter values that demonstrated convergence of the algorithm to a consensus state with frequency-locked oscillators.

The preliminary results reported in this paper show that consensus theory provides a powerful new approach to the problem of carrier synchronization. This opens up many exciting open problems for future work. An analytical characterization of the properties of the proposed algorithm is one such topic for further study. Implementing consensus-based algorithms on recently-developed experimental prototypes for cooperative communication schemes is another interesting open topic.

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